Limite remarcabile

Tabelul limitelor fundamentale, operații cu ∞, cazuri de nedeterminare și regula lui l'Hôpital **matepenet.ro**

Limita remarcabilă	Forma generală: $u = u(x) \to 0$, $u \neq 0$	Valoare
$\lim\limits_{x\to 0} \dfrac{\sin x}{x}$	$\lim\limits_{u\to 0} \dfrac{\sin u}{u}$	1
$\lim\limits_{x\to 0} \dfrac{\operatorname{tg} x}{x}$	$\lim\limits_{u\to 0} \dfrac{\operatorname{tg} u}{u}$	1
$\lim\limits_{x\to 0} \dfrac{\arcsin x}{x}$	$\lim\limits_{u\to 0} \dfrac{\arcsin u}{u}$	1
$\lim\limits_{x\to 0} \dfrac{\operatorname{arctg} x}{x}$	$\lim\limits_{u\to 0} \dfrac{\operatorname{arctg} u}{u}$	1
$\lim\limits_{x\to 0} \dfrac{1-\cos x}{x^2}$	$\lim\limits_{u\to 0} \dfrac{1-\cos u}{u^2}$	$\dfrac{1}{2}$
$\lim\limits_{x\to 0}(1+x)^{\frac{1}{x}}$	$\lim\limits_{u\to 0}(1+u)^{\frac{1}{u}}$	e
$\lim\limits_{x\to\pm\infty}\left(1+\dfrac{1}{x}\right)^{x}$	$\lim\limits_{u\to\pm\infty}\left(1+\dfrac{1}{u}\right)^{u}$	e
$\lim\limits_{x\to 0} \dfrac{\ln(1+x)}{x}$	$\lim\limits_{u\to 0} \dfrac{\ln(1+u)}{u}$	1
$\lim\limits_{x\to 0} \dfrac{\log_a(1+x)}{x}$	$\lim\limits_{u\to 0} \dfrac{\log_a(1+u)}{u}$	$\dfrac{1}{\ln a}$
$\lim\limits_{x\to 0} \dfrac{e^x-1}{x}$	$\lim\limits_{u\to 0} \dfrac{e^u-1}{u}$	1
$\lim\limits_{x\to 0} \dfrac{a^x-1}{x}$	$\lim\limits_{u\to 0} \dfrac{a^u-1}{u}$	$\ln a$
$\lim\limits_{x\to 0} \dfrac{(1+x)^r-1}{x}$	$\lim\limits_{u\to 0} \dfrac{(1+u)^r-1}{u}$	r

$a > 0$ (la $\log_a$ și $a \neq 1$); $r \in \mathbb{R}$. Argumentul și numitorul trebuie să fie aceeași expresie: $\lim\limits_{x\to 0} \dfrac{\sin 3x}{x} = \lim\limits_{x\to 0} \dfrac{\sin 3x}{3x} \cdot 3 = 3$.

Operații cu ∞ ($a \in \mathbb{R}$)

$a \pm \infty = \pm\infty$, $\infty + \infty = \infty$, $\infty \cdot \infty = \infty$

$a \cdot \infty = \infty$ dacă $a > 0$; $= -\infty$ dacă $a < 0$

$\dfrac{a}{\pm\infty} = 0$, $\dfrac{1}{0^+} = +\infty$, $\dfrac{1}{0^-} = -\infty$

$e^{\infty} = \infty$, $e^{-\infty} = 0$, $\ln \infty = \infty$, $\ln 0^+ = -\infty$

$\operatorname{arctg}(\pm\infty) = \pm\frac{\pi}{2}$; $\operatorname{arcctg}(+\infty) = 0$, $\operatorname{arcctg}(-\infty) = \pi$

$a^{\infty} = \infty$ pt. $a > 1$; $a^{\infty} = 0$ pt. $0 < a < 1$

Cazuri de nedeterminare

$\dfrac{0}{0}$, $\dfrac{\infty}{\infty}$, $\infty - \infty$, $0 \cdot \infty$, 1^{∞}, 0^0, ∞^0

1^{∞}: $\lim u^v = e^{\lim (u-1)v}$, pentru $u \to 1$, $v \to \infty$

0^0, ∞^0: $u^v = e^{v \ln u}$

Funcții raționale la $\pm\infty$

$$\lim_{x\to\infty} \frac{a_n x^n + \ldots}{b_m x^m + \ldots} = \begin{cases} 0, & n < m \\ \frac{a_n}{b_m}, & n = m \\ \pm\infty, & n > m \end{cases}$$

Regula lui l'Hôpital

Pentru $\frac{0}{0}$ sau $\frac{\infty}{\infty}$, dacă există $\lim\limits_{x\to x_0} \dfrac{f'(x)}{g'(x)}$:

$$\lim_{x\to x_0} \frac{f(x)}{g(x)} = \lim_{x\to x_0} \frac{f'(x)}{g'(x)}$$

Alte limite utile

$\lim\limits_{x\to\infty} \dfrac{x^n}{e^x} = 0$, $\lim\limits_{x\to\infty} \dfrac{\ln x}{x^k} = 0$ ($k > 0$)

$\lim\limits_{x\searrow 0} x \ln x = 0$, $\lim\limits_{x\to\infty} \dfrac{\sin x}{x} = 0$